ANALYSIS OF A KERNEL-BASED METHOD FOR SOLVING VOLTERRA-FREDHOLM INTEGRAL EQUATIONS

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ABSTRACT

In this paper, we investigate Volterra-Fredholm integral equations (VFIEs) by a numerical method based on reproducing kernels. We derive some effective error estimates for the proposed method when applied to VFIEs. Numerical experiment is carried out to illustrate the efficiency and applicability of the suggested method.
Keywords: Volterra-Fredholm; Reproducing Kernel; Error estimate.

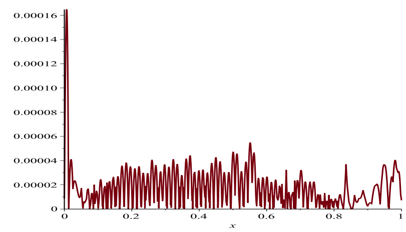

Figure 1: The plot of absolute error for Example 3.1.

Theorem 2.3. $\vartheta_i(x) = Ł_t R_x^{\langle m \rangle}(t)|_{t=x_i}, i = 1,2,\cdots,$ *where the subscript t in the operator Ł indicates that the operator Ł applies to the function of t.*

Lemma 2.4. *Let* $\{x_i\}_{i=1}^{\infty}$ *be a countable dense subset in the domain* $[a,b]$*, then* $\{\vartheta_i(x)\}_{i=1}^{\infty}$ *is a complete system of the space* W_2^m*.*

Usually, a normalized orthogonal system is constructed from $\{\vartheta_i(x)\}_{i=1}^{\infty}$ by using the Gram-Schmidt process. Now, suppose that $\{\overline{\vartheta}_i(x)\}_{i=1}^{\infty}$ be a normalized orthogonal system in W_2^m, such that $\overline{\vartheta}_i(x) = \sum_{k=1}^{i} \delta_{ik}\vartheta_k(x),\ i = 1,2,\cdots$, then we state the following theorem that gives the exact solution.

Theorem 2.5. *Let* $\{x_i\}_{i=1}^{\infty}$ *be dense in* $[a,b]$ *and the solution of (2.2) be unique, then the exact solution of (2.2) will be*

$$y(x) = \sum_{i=1}^{\infty}\sum_{k=1}^{i} \delta_{ik} f(x_k)\overline{\vartheta}_i(x). \tag{2.5}$$

Then the approximate solution can be obtained by calculating a truncated series based on these functions as the following

$$y_n(x) = \sum_{i=1}^{n} \beta_i \overline{\vartheta}_i(x) = \sum_{i=1}^{n}\sum_{k=1}^{i} \delta_{ik} f(x_k)\overline{\vartheta}_i(x). \tag{2.6}$$

Lemma 2.6. *For any* $y \in W_2^m[a,b]$ *we have the following statement*

$$\|y^{(k)}\|_\infty \le c_k\ \|y\|_{W_2^m},\ k = 1,2,...,m-1, \tag{2.7}$$

for some c_k *independent of x.*

Theorem 2.7. ***(Convergence Theorem)*** *Let* $\{x_i\}_{i=1}^{\infty}$ *be dense in* $[a,b]$ *and y is the solution of (1.1)-(**??**), then the approximate solution* y_n *and its derivative* $y_n^{(k)}$*,* $k = 1,2,...,m-1$*, are uniformly convergent to the exact solution y and* $y^{(k)}$*, respectively.*

Theorem 2.8. *If* y_n *be the approximate solution of Eq. (1.1) in the space* $W_2^2[a,b]$*. Then the following relation holds,*

$$\|y^{(k)} - y_n^{(k)}\|_\infty \le ch,\ k = 0,1,\ h = \max_{1\le i\le n-1}(x_{i+1} - x_i), \tag{2.8}$$

where c is real constant.

Theorem 2.9. *If* y_n *be the approximate solution of Eq. (1.1) in the space* $W_2^3[a,b]$*. Then the following relation holds,*

$$\|y^{(k)} - y_n^{(k)}\|_\infty \le ch^3, k = 0,1,2, \tag{2.9}$$

where c is a real constant.

3. TEST EXAMPLE

Example 3.1. *Consider*

$$h(x) = x^2, k_1(x,t) = e^{x-t}, k_2(x,t) = e^{x+t}, \lambda_1 = 1, \lambda_2 = -1, a = 0, b = 1.$$

We choose $f(x)$ *such that VFIE has exact solution* $y(x) = x^2$*.*

4. REFERENCES

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